**2D FDTD with PML (working Title)**

*Authors: Adedayo Lawal and Blake Levy*

**Abstract – A two dimensional finite difference time domain (FDTD) simulation is presented. The computational domain is surrounded by a perfectly matched layer (PML) which is terminated by a perfect electric conductor (PEC).**

1. **INTRODUCTION**
2. **FORMULATION**
   1. **Yee Cell**

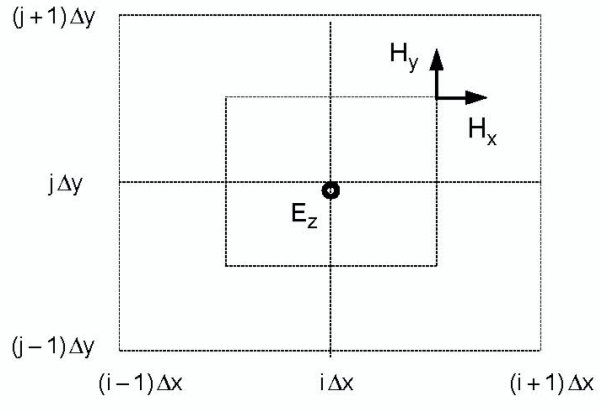
In order to simulate the electric and magnetic fields in a 2-Dimensional geometry, a conventional Yee Cell method was used. Figure 1 shows the half-step offset of the magnetic field grid related to the electric field grid.

Figure 1 Staggered Grid1

The project defined a current source that was oriented in the plane perpendicular to the field grid. As a result of this current source, the only nonzero electric field component was in the direction of the current source, , whereas the nonzero magnetic field components corresponded to and which indicated transverse magnetic (TM) field behavior. Equations (2.1a)—(2.1c) are derived from Faraday and Ampere’s law in a Cartesian two-dimensional geometry based upon our source-free, non-zero electric and magnetic field components.

By applying a second order accurate central difference method to the above equations, update equations for , , and are determined. The “leap-frog” technique can be used to find the latest values of based off of the latest values of and .

* 1. **Dispersion Relation**

The update equations used in the “leap-frog” technique can also provide a necessary condition that relates the time-step, , to the space discretization of the computational domain, and . Assume the field components are plane waves propagating at a certain angle with respect to the positive y-axis, the wavenumber can then be defined as where and . By relating the update equations we can come up with a dispersion relation in Equation (2.2.1) and this can be used to put an upper limit on the time-step.

If and we note that the terms maximum value is one, then .

* 1. **Permittivity Discontinuity**

The geometry used in this project contains a dielectric half-space, resulting in a discontinuity of permittivity in the z-direction. In order to enforce Equation (2.1.1a), we must revert to Ampere’s law and enforce it at the interface. Figure 2 shows the unit cell that is used to evaluate Ampere’s law in integral form. Equation (2.3.1) can now be used for the update equation (2.1.1a) with an appropriate value of permittivity.

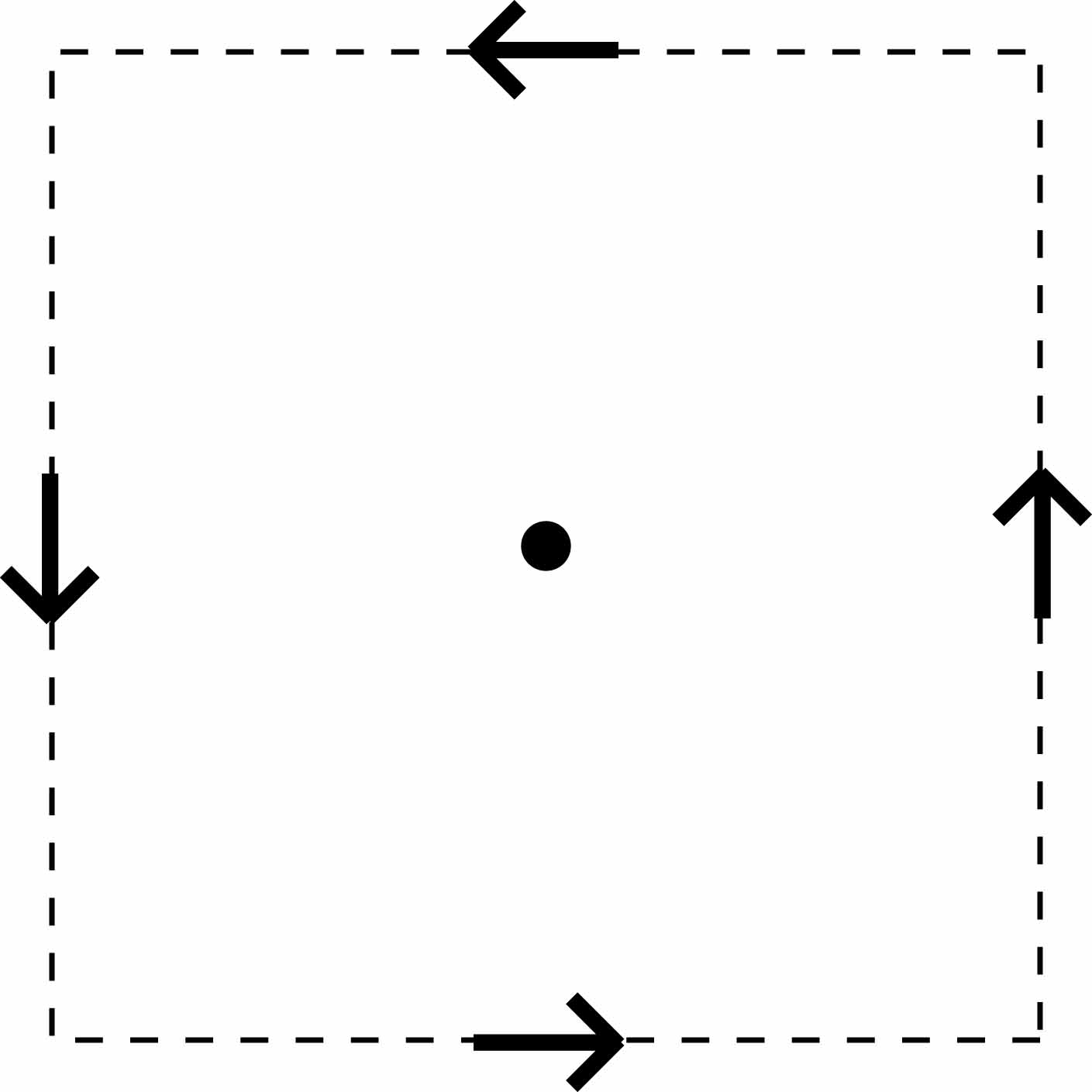
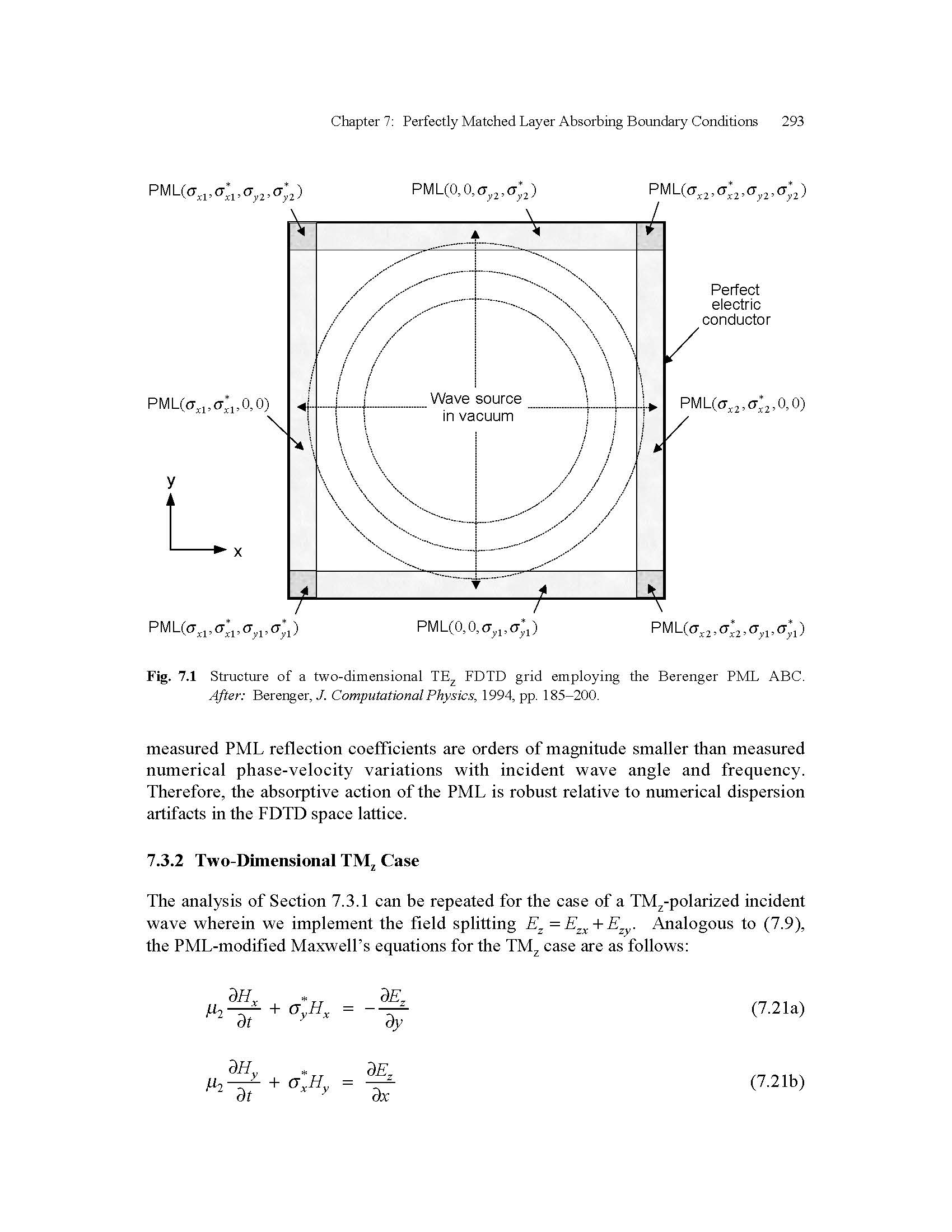


Figure 2 Unit Cell

* 1. **Perfectly Matched Layer (PML)**

1. **RESULTS**
2. **CONCLUSION**
3. **REFERENCES**
4. Kane Yee, "Numerical solution of initial boundary value problems involving maxwell's equations in isotropic media," *Antennas and Propagation, IEEE Transactions on* , vol.14, no.3, pp.302,307, May 1966  
   doi: 10.1109/TAP.1966.1138693
5. Taflove, Allen, and Susan C. Hagness. *Computational Electrodynamics: The Finite-difference Time-domain Method*. Boston: Artech House, 2005. Print.
6. Berenger, J. P., “A perfectly matched layer for the absorption of electromagnetic waves,” *J. Computational Physics*, Vol. 114, 1994, pp. 185-200.