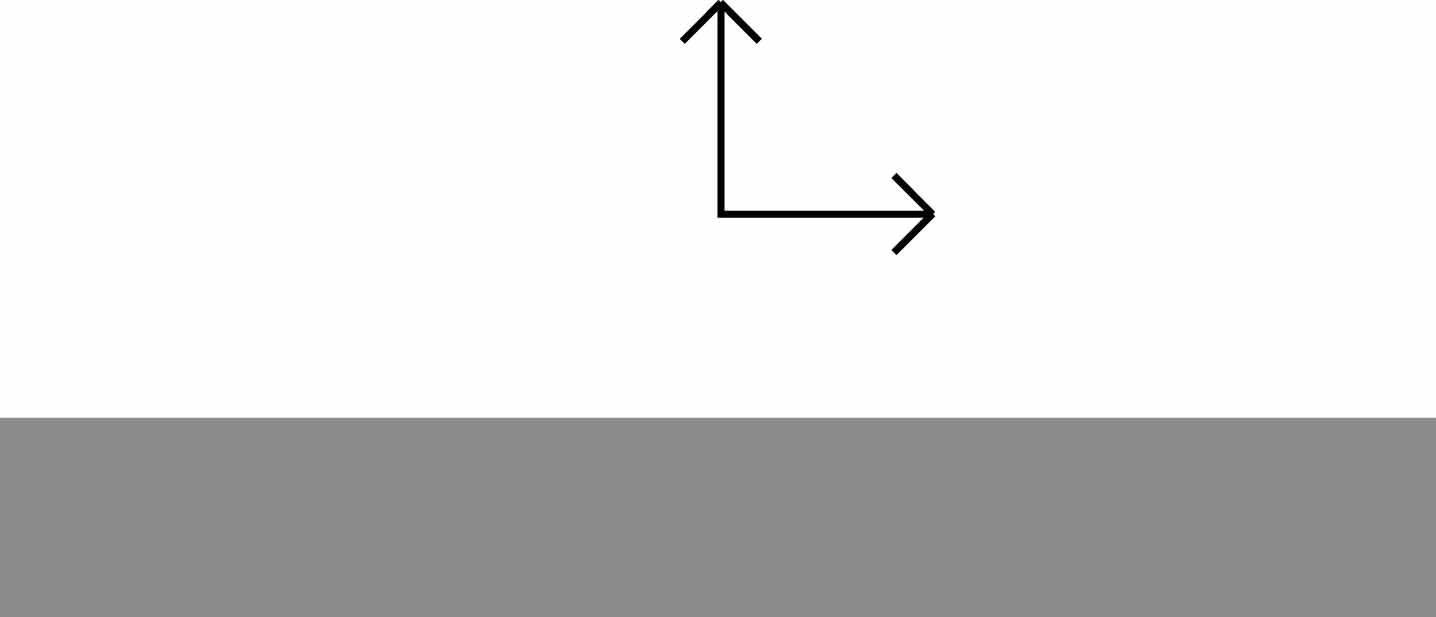
**2D FDTD with PML (working Title)**

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**Abstract – A two dimensional finite difference time domain (FDTD) simulation is presented. The computational domain is surrounded by a perfectly matched layer (PML) which is terminated by a perfect electric conductor (PEC).**

1. **INTRODUCTION**

Many real world electromagnetic problems can be simplified by the elimination of one spatial dimension. For the problems that do not result in a closed form analytical solution, computational electromagnetics can offer an understanding of the field properties that exist in space. Finite Difference Time Domain (FDTD) is one way to provide a solution for these types of problems. In the following paper, formulations along with results are provided to solve for the fields of an X-directed current source radiating into a YZ-plane as shown in Figure 1.



y

z

Figure 1 Project Geometry

1. **FORMULATION**
   1. **Yee Cell**

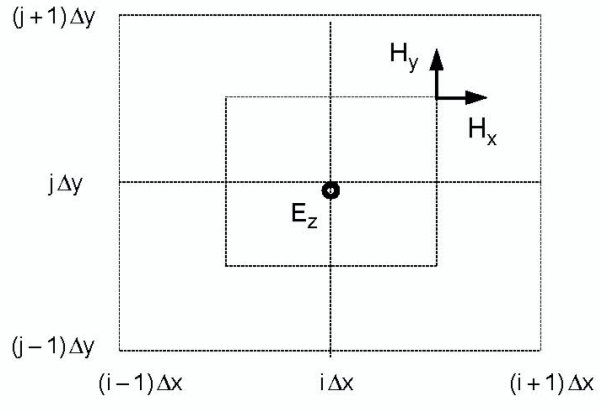
In order to simulate the electric and magnetic fields in a 2-Dimensional geometry, a conventional Yee Cell method was used. Figure 1 shows the half-step offset of the magnetic field grid related to the electric field grid.

Figure 2 Staggered Grid1

The project defined a current source that was oriented in the plane perpendicular to the field grid. As a result of this current source, the only nonzero electric field component was in the direction of the current source, , whereas the nonzero magnetic field components corresponded to and which indicated transverse magnetic (TM) field behavior. Equations (2.1a)—(2.1c) are derived from Faraday and Ampere’s law in a Cartesian two-dimensional geometry based upon our source-free, non-zero electric and magnetic field components.

By applying a second order accurate central difference method to the above equations, update equations for , , and are determined. The “leap-frog” technique can be used to find the latest values of based off of the latest values of and .

* 1. **Dispersion Relation**

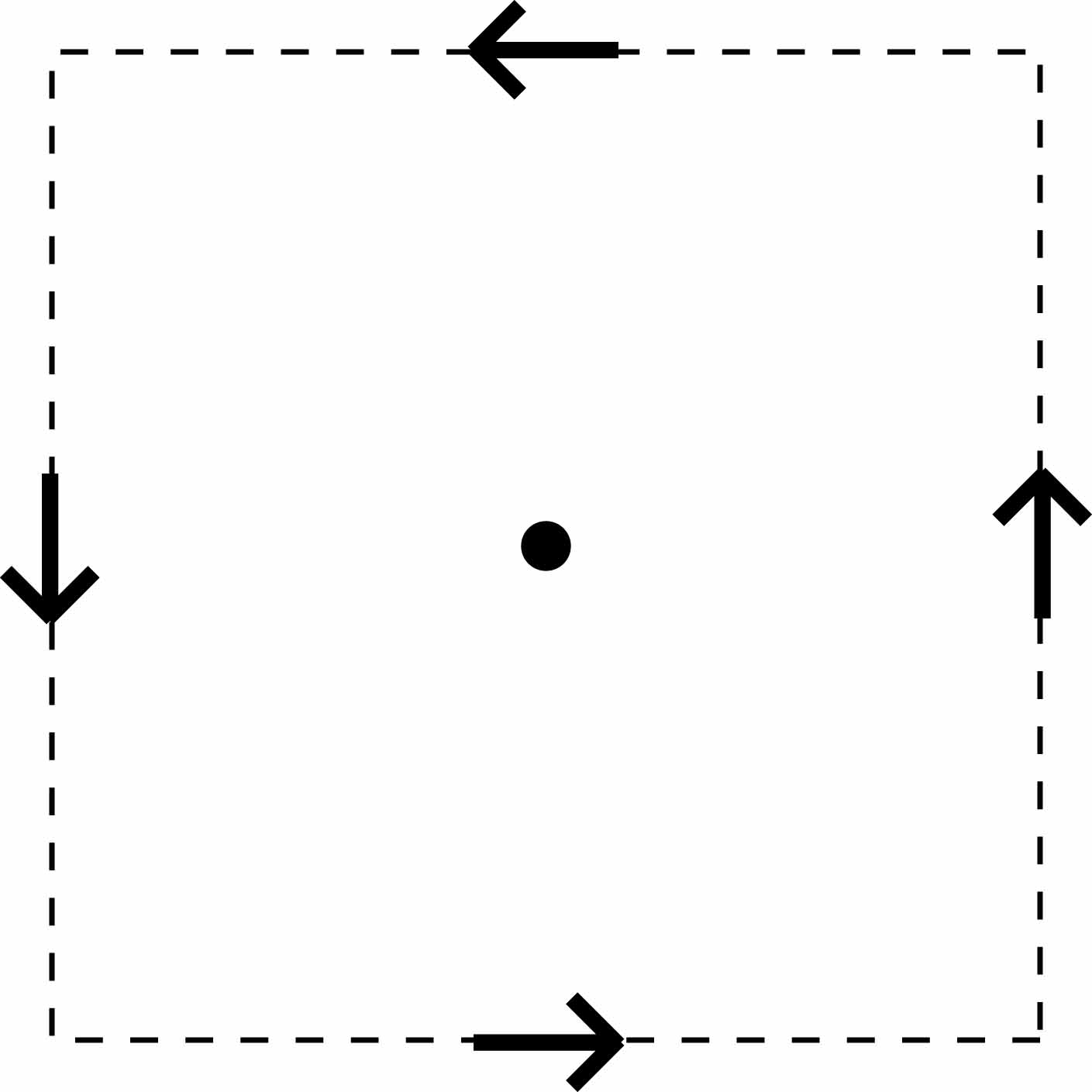
The update equations used in the “leap-frog” technique can also provide a necessary condition that relates the time-step, , to the space discretization of the computational domain, and . Assume the field components are plane waves propagating at a certain angle with respect to the positive y-axis, the wavenumber can then be defined as where and . By relating the update equations we can come up with a dispersion relation in Equation (2.2.1) and this can be used to put an upper limit on the time-step.

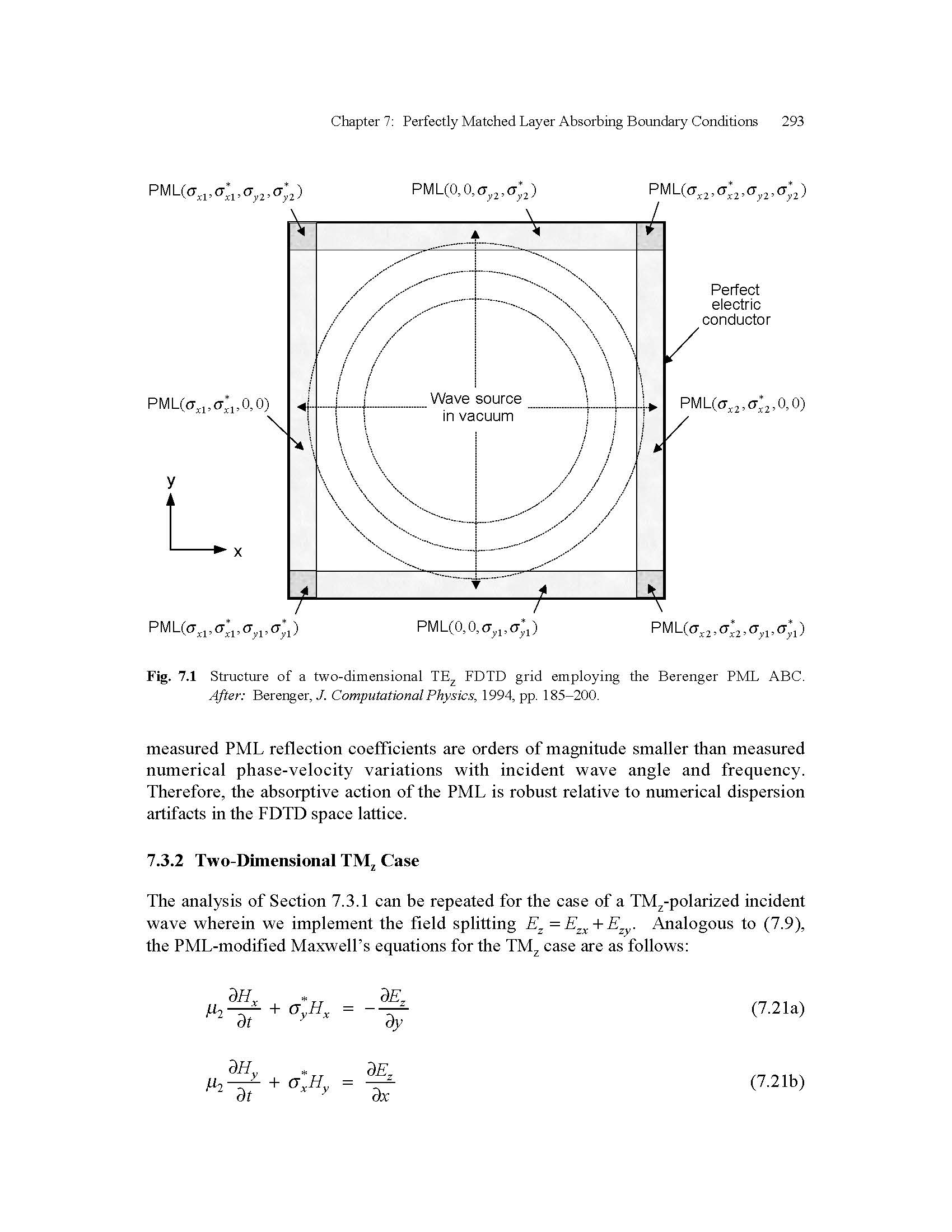
If and we note that the terms maximum value is one, then

* 1. **Permittivity Discontinuity**

The geometry used in this project contains a dielectric half-space, resulting in a discontinuity of permittivity in the z-direction. In order to enforce Equation (2.1.1a), we must revert to Ampere’s law and enforce it at the interface. Figure 2 shows the unit cell that is used to evaluate Ampere’s law in integral form. Equation (2.3.1) can now be used for the update equation (2.1.1a) with an appropriate value of permittivity.

Figure 3 Unit Cell

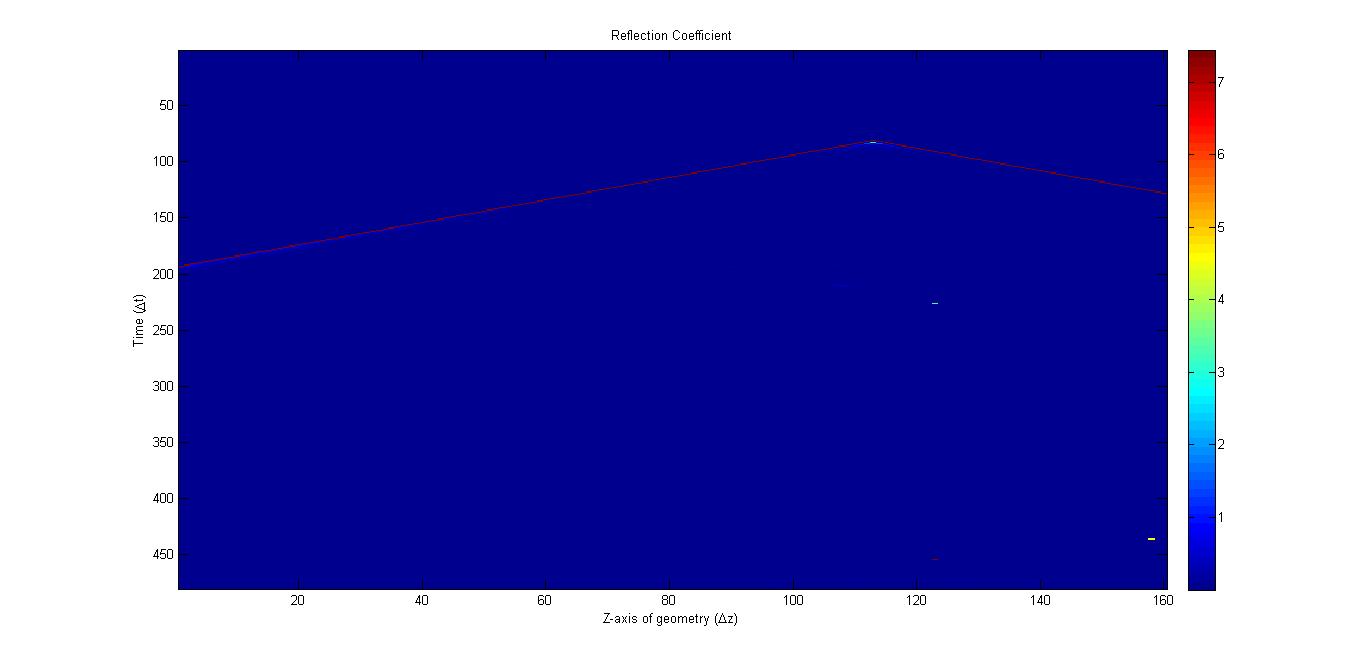


* 1. **Perfectly Matched Layer (PML)**

1. **RESULTS**
   1. **Use of PML**

Upon implementation of the perfectly matched layer, the interface between the dielectric region of the geometry and PML interface yielded reflections as high as 33%. In order to mitigate these reflections, a graded PML was used. Equation (3.1.1) determined the conductivity inside the PML regions surrounding the computational domain. By experimentation, reflections on the order of <1% were found with a value of and with the PML width of 20 grid cells. Figure 4a. compares the electric field with and without a perfectly matched layer between a PEC boundary.

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To verify the operation of the PML, we can look at the electric field two the left and right of the PML. If there are any reflections at the PML interface, they will propagate back into the computational domain. We can quantify these reflections by Equation (3.1.2). Figure 4b. shows the reflection coefficient of the left PML boundary where almost all points are colored blue which corresponds to a value of approximately .3%. The vertex indicates where the wave has impinged on the z-axis of the boundary and the red line indicates the “light-line” or speed at which the wave moves along the boundary. The small spikes may indicate computational errors.

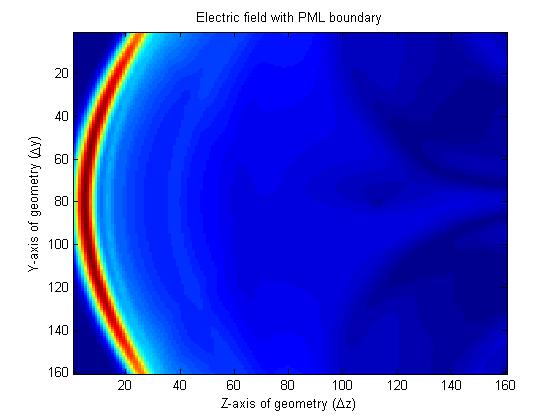
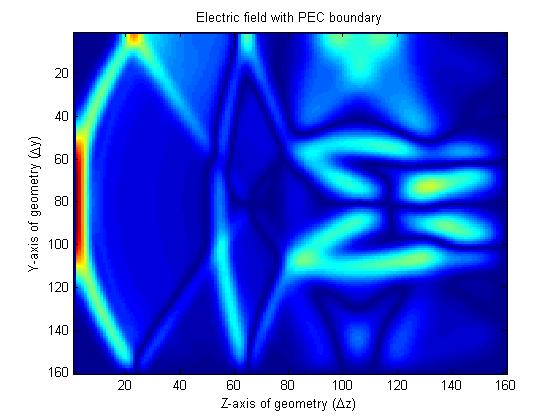


Figure 4 1 a) Left: Electric field with PEC boundary. Right: Electric field with PML boundary Bottom: Reflection Coefficient

1. **CONCLUSION**
2. **REFERENCES**
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